4 MICHELSON
This session:

- The Michelson interferometer, the core of LIGO
- How does it work?
- Why the Michelson? And why the 'dark fringe'?
Use coupling equations

\[ a_2 = \sqrt{\frac{1}{2}} a_1, \quad a_3 = a_2 e^{-i2\pi k L_1} \]

\[ a_4 = i\sqrt{\frac{1}{2}} a_1, \quad a_5 = a_4 e^{-i2\pi k L_2} \]

\[ a_6 = i\sqrt{\frac{1}{2}} a_3 + \sqrt{\frac{1}{2}} a_5 = i\frac{1}{2} e^{-i2\pi k L_1} a_1 \]

\[ + i\frac{1}{2} e^{-i2\pi k L_2} a_1 = i\frac{1}{2} a_1 (e^{-i2\pi k L_1} + e^{-i2\pi k L_2}) \]

\[ a_7 = \frac{1}{2} a_3 + i\sqrt{\frac{1}{2}} a_5 \]

Then

\[ a_6 = a_1 \frac{i}{2} e^{-i2\pi k L} \left( e^{-i2\pi k DL} + e^{i2\pi k DL} \right) \]

\[ = a_1 i e^{-i2\pi k L} \cos(2\pi k DL) \]

We can simplify using

\[ \bar{L} = \frac{1}{2} (L_1 + L_2) \]

\[ \Delta L = \frac{1}{2} (L_1 - L_2) \]

\[ \rightarrow \bar{L} + \Delta L = L_1 \]

\[ \bar{L} - \Delta L = L_2 \]

\[ \Delta L \text{ changes amplitude} \]

\[ \bar{L} \text{ changes phase} \]
Power in output

\[ P_6 = P_1 \cdot \cos^2(2k \Delta L) \]

Minimum at \( 2k \Delta L = (2N+1) \frac{\pi}{2} \)

\[ \Rightarrow \Delta L = (2N+1) \frac{\lambda}{8} \]

Operating points

- Want cavity with large circulating power \( \Rightarrow \) on resonance
- Want Michelson to be on dark fringe \( \text{(Why? Thursday!)} \)

Mirror positions and distance between objects can vary but interferometers only work (well) if positions and distances are well defined.

\[ \Rightarrow \text{On the simulation, need to chase 'tuning' of mirrors carefully} \]

In the experiment, control systems constantly measure and correct positions
Degrees of freedom (DoF)

Michelson output: $a_6 = i a_1 e^{-i 2 \mu \epsilon} \cos (2 \mu \Delta L)$

Chose to describe Michelson with $E$, $\Delta L$ instead of $L_1$, $L_2$ because this highlights the behaviour of the instrument better.

$E$ changes phase, $\Delta L$ changes amplitude, while $L_1$ or $L_2$ could change both.

These new variables are the degrees of freedom.

$E$ common mode, $\Delta L$ differential mode.

Important for all multi-optics systems, example coupled cavity.

To investigate move $m_1$ or $m_3$ but not $m_2$!
Length signal and frequency noise

We want to measure length. $L$ always appears as $\cos(2kL)$. 

\[ k = \frac{\omega}{c} = \frac{2\pi f}{c} \], \[ \lambda = 10^{-6} \], \[ c = \frac{1}{A} \] \[ A = 3 \times 10^{14} \]

Assume \( L = L_0 + SL \), \[ \delta L = \delta L_0 + \delta SL \]

constant signal  \quad constant noise

Cavity (just look at $2kL$):

\[ 2kL = \frac{4\pi}{c} \int L \]

\[ \frac{4\pi}{c} \left( \delta L_0 + \delta SL + \delta SL_0 + \delta SL_0 \right) \approx \frac{4\pi}{c} \left( \delta SL + \delta SL_0 \right) \]

\[ \frac{4\pi}{c} \delta L_0 = N \cdot 2\pi \text{ on resonance} \]

We need \[ \delta SL > \delta SL_0 \Rightarrow \left( \frac{SL}{L_0} \right) > \left( \frac{SL_0}{L_0} \right) \]

\[ 10^{-23} \]

\[ 10^{-15} \text{ for a very good laser!} \]
Same for Nichelson

\[ L = L_0 + \Delta L + SL \]

Power \( \sim \cos(2\pi \Delta L) \)

\[ 2\pi \Delta L = \frac{4\pi}{c} (\delta_0 + \delta_f) (\Delta L + SL) = \frac{4\pi}{c} (\delta_0 \Delta L + \delta_0 SL + \delta_f \Delta L + \delta_f SL) \]

Now: \[ \frac{\delta_f}{\delta_0} < \frac{SL}{\Delta L} \, , \, SL \approx 10^{-2} m \Rightarrow \frac{\delta_f}{\delta_0} < 10^{-17} \]

6 orders of magnitude better than a cavity (and still difficult!)

Main reason for using a Nichelson.
Why a Michelson?

- an interferometer scales signal by optical frequency $f_0 = 10^{14}$!
- measures GW signal 'twice'

- Michelson is many orders of magnitude (6!) less sensitive to freq noise

\[ \begin{align*}
L_2 \\
L_1
\end{align*} \quad \begin{align*}
\Delta L
\end{align*} \]
Signal gradient

\[ \frac{dP}{d\Delta L} = -8P_k \sin(4\pi \Delta L) \]

Best 'sensitivity' at mid fringe.

Why use dark fringe?

1) Slope misleading. Sensitivity really depends on signal vs. noise, e.g. mirror motion noise or laser freq. noise scale the same.

2) Will see on Thursday that dark fringes will allow us to do a trick and gain laser power (for quantum noise reduction).

3) Generally good to do 'null experiments'. At half range 250W on photo diode.
| **Michelson with Power Recycling** |
| **Michelson** |
| **Hach-Zehnder** |
| **Triangular ring cavity** |
| **Ring cavity** |
Summary:

- equations for Michelson output
- Michelson has 2 degrees of freedom (common + differential)
- Michelson much less sensitive to laser frequency noise
- operate Michelson at 'dark fringe' (see Thursday)

Tomorrow:

- modulation of fields
- signals vs noise → sensitivity
- transfer functions